DETERMINATION OF BASE PRESSURE BEHIND A STEP IN SEPARATED FLOW

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We assume the problem of determining the base pressure behind a step in the presence of turbulent mixing, with and without allowance for the initial boundary-layer thickness, in a plane-parallel supersonic stream. A comparative estimate of base pressure is made with various boundary conditions for open and closed stagnant zones.

A scheme for calculating the base pressure of a compressible gas behind a step has been formulated by Korst [1], based on the Tollmien problem of mixing of a semi-infinite jet with a gas at rest occupying a half-space. To determine the velocity profile, Korst, assuming the flow in the mixing layer to be self-similar, used the solution of the approximate parabolic equation of motion in the boundary layer

$$u\frac{\partial u}{\partial x} = \varepsilon\,\frac{\partial^2 u}{\partial y^2},$$

and to determine the circulation in the stagnant zone—the condition of branching in the junction region of the separated flow (Korst-Chapman conditions): the total pressure P_{0j} on the branch line is equal to P_{4a} in the zero-gradient part of the flow behind the oblique shock.

As has been shown by Nash [2], the attachment pressure, and correspondingly the total pressure on the branch line, are not equal to the maximum static pressure realized upon attachment of the separated zero-gradient part of the stream, but are considerably smaller. Moreover, the initial boundary layer at the point of separation of the oncoming stream has a considerable influence on the value of the base pressure.

The present article gives the results of calculations of base pressure during turbulent mixing of a plane stream behind a projection for the "exact" equation of motion

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y},$$

with $Pr_T = 1$ and $T_0 = \text{const}$. Solution of the equations of turbulent mixing without allowance for initial boundary-layer thickness simplifies the problem appreciably and allows a comparatively simple determination of the influence of mass flow rate of the gas issuing from the base region, and of the influence of the quantities $\chi = C_p/C_V$ and σ (σ is a parameter describing the propagation of turbulence in the mixing zone) on the magnitude and nature of the change of base pressure. As initial parameters we used the results of calculations of self-similar turbulent mixing behind a step of a semi-infinite jet and a motionless gas [3].

Flow model: To calculate the base pressure behind a step we assumed the following flow model of a plane stream with four characteristic regions (Fig. 1): region of uniform approaching stream (1), region of deflected uniform stream, i.e., Prandtl-Meyer flow (2), region of separated flow with adjoining turbulent mixing zone (3), and region of attachment of separated stream with an isentropic pressure increase (4).

In accordance with the Korst-Chapman hypothesis, it has been assumed that the streamline dividing the circulating mass in the stagnant zone and the external stream is characterized by a total pressure equal to the static pressure in the pressure increase region ($P_{0j} = P_{4a}$).

Determination of base pressure for a closed stagnant zone. 1. No allowance for initial boundary layer thickness. We shall denote by u_j and P_{0j}, respectively, the velocity component and the total pressure on the branch line; then

$$\frac{P_b}{P_{0j}} = \left[1 - \frac{\varkappa-1}{\varkappa+1}\left(\frac{u_j}{a_{cr}}\right)^2\right]^{\frac{\varkappa}{\varkappa-2}} =$$

$$= \left[1 + \frac{\varkappa-1}{2}\left(1-\bar{u}_j^2\right) M_{3a}^2\right]^{\frac{\varkappa}{\varkappa-1}} \Big/ \Big[1 +$$

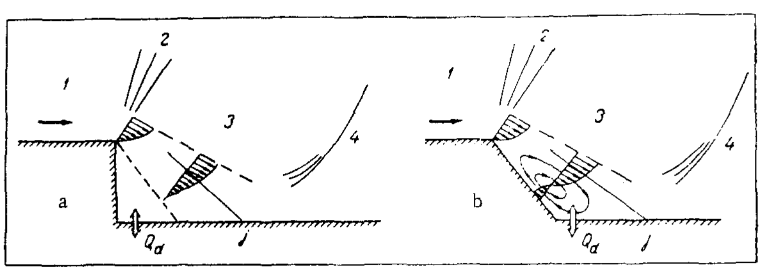

Fig. 1. Flow model of a plane stream behind a step with the boundary conditions of the Tollmien (a) and Dem'yanov-Shmanenkov (b) problems.

$$\frac{\varkappa-1}{2}\mathrm{M}_{3a}^{2}\Big]^{\frac{\varkappa}{\varkappa-1}}=\frac{\pi(\mathrm{M}_{3a})}{\pi(\overline{\mathrm{M}}_{3a})}, \quad (1)$$

where $\overline{\mathrm{M}}_{3a}=\mathrm{M}_{3a}\sqrt{1-\bar{u}_j^{2}}$.

Assuming that at the end of the separated zone the static pressure of the oncoming stream $P_{4a} \parallel P_{1a}$ is achieved, we obtain a simple approximate solution for determining the base pressure

$$P_b/P_{1a}=\pi\left(\mathrm{M}_{1a}\Big/\sqrt{1-\bar{u}_j^{2}}\right)\Big/\pi(\mathrm{M}_{1a}). \quad (2)$$

Values of dimensionless velocity $\bar{u}_j$ are determined according to [3] as a function of the Crocco number Cr_{3a} for the boundary conditions of the Tollmien and Dem'yanov-Shmanenkov problems. The results of the calculations are shown in Fig. 2, as are calculated values of base pressure obtained using the semi-empirical relation $P_{0j} = 0.35(P_{1a} - P_b) + P_b$, proposed by Nash [2].

The different boundary conditions in the turbulent mixing zone (see Fig. 1 and [3]) for the Tollmien and Dem'yanov-Shmanenkov problems lead to different velocity profiles in the mixing zone. The boundary conditions of the Dem yanov-Shmanenkov problem give a velocity profile with smaller $\bar{u}_j$ on the branch line than in the Tollmien boundary problem.

2. Allowance for initial boundary layer thickness. The initial boundary layer thickness at the separation point proves to have a substantial influence on the turbulent mixing zone. There is an initial section of the turbulent mixing zone ($x \le x_*$) in which there is still no self-similarity of the velocity profile. The absence of self-similarity leads to a change in the value of $u_j = F'_{F=0}$. A determination was made in [3] of the influence of momentum thickness δ_2^{**}/x of the boundary layer on the quantity $\bar{u}_j = F'_{F=0}$ at various Crocco numbers Cr_{3a} for the Tollmien boundary problem. With the Nash [2] relation $P_{0j} = 0.35(P_{1a} - P_b) + P_b$ and the relation

$$\frac{P_b}{P_{0j}}=\left[1-\frac{\varkappa-1}{\varkappa+1}\lambda_j^{2}\right]^{\frac{\varkappa}{\varkappa-1}}, \quad (3)$$

we find λ_j on the branch line, after assigning P_b/P_{1a}, for given values of M_{1a} and $\varkappa$. From the relation

$$P_b/P_{1a}=\pi(\lambda_{1a})/\pi(\lambda_{3a}) \quad (4)$$

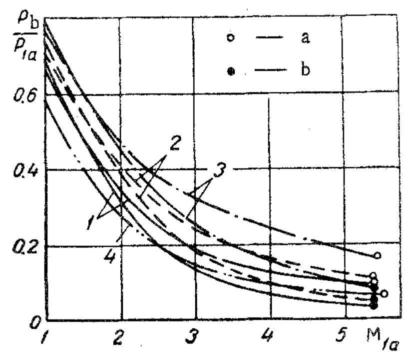

Fig. 2. Dependence of relative base pressure P_b/P_{1a} on M_{1a} with $\varkappa = 1.4$ (*a*) and 1.2 (b) for boundary conditions of the Tollmien problem in the Korst solution (1), and the Neiland solution (2), of the Dem'yanov-Shmanenkov problem (3) and the Tollmien problems in the Neiland solution with the Nash condition $P_{0j} = 0.35(P_{1a} - P_b) + P_b$ (4).

we determine $F'_{F=0} = \lambda_j/\lambda_{3a}$. A specific value of δ_2^{**}/x at the beginning of the mixing zone corresponds to the values of $F'_{F=0}$ and Cr_{3a} found. To determine the initial boundary-layer momentum thickness at the

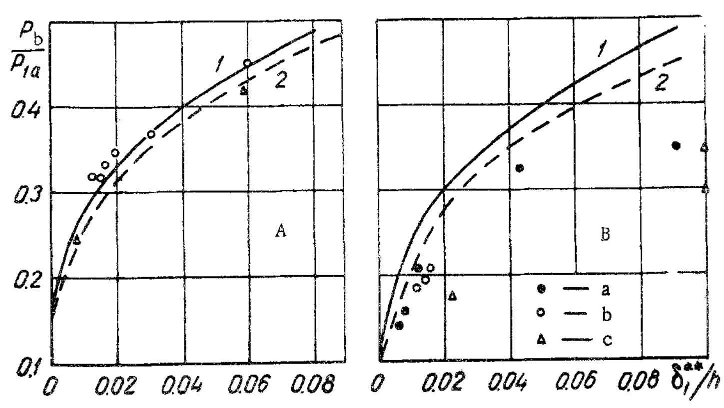

Fig. 3. Dependence of relative base pressure P_b/P_{1a} on the the initial boundary-layer momentum thickness $\delta_1^{**}h$ at the separation point, with $M_{1a} = 2$ (A) and (B), $\varkappa = 1.4$: 1) according to Nash [2]; 2) according to our calculation; a, b, c) experimental data of various authors [2].

separation point up to turning of the stream, the relation proposed by Nash [2] was used:

$$\rho_{3a} u_{3a} \delta_2^{**}/\rho_{1a} u_{1a} \delta_1^{**} = M_{1a}^2/M_{3a}^2. \quad (5)$$

The results of the calculations are shown in Fig. 3.

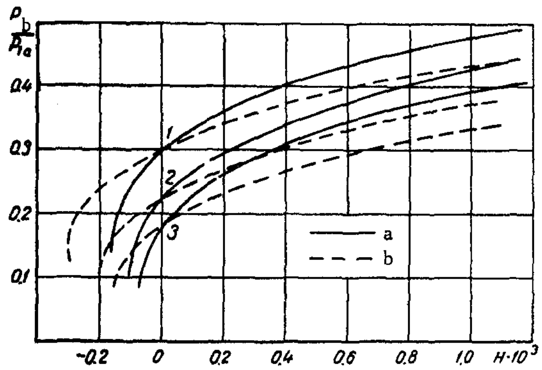

Fig. 4. Dependence of relative base pressure P_b/P_{1a} on dimensionless flow rate H of gas, with $M_{1a} = 2.76$, $\varkappa = 1.2$, for boundary conditions of the Dem'yanov-Shmanenkov problem (1), the Tollmien problem in the Neiland solution (2), and in the Korst solution (3): a) with $\sigma = 12 + 2.578_{3a}$; b) with $\sigma = 12$.

Determination of base pressure for an open stagnant zone without allowance for initial boundary layer thickness. The base pressure for an open stagnant zone was determined with the same conditions as for the closed stagnant zone. Following Korst [1] we shall denote by $H = Q_d/Q_{1a}^*$ the dimensionless flow rate of gas, where $Q_d = \zeta\rho_{3a} 1/\sigma\, x_* u_{3a} F$ is the mass flow rate of gas per second, and $Q_{1a}^* = \frac{P_{01a}}{\sqrt{T_{01a}}}\sqrt{\frac{\varkappa\zeta}{R}}\, h$ is the relative mass flow rate per second of the oncoming stream. Then

$$H = \zeta\rho_{3a}\frac{1}{\sigma}x_* u_{3a}F \Big/ \frac{P_{01a}}{\sqrt{T_{01a}}}\sqrt{\frac{\varkappa\zeta}{R}}\,h = \frac{P_{3a}}{P_{01a}}\sqrt{\frac{T_{01a}}{T_{3a}}}\frac{1}{\sigma}\frac{M_{3a}}{\sin\Theta_b}F. \quad (6)$$

Notation

u, v—longitudinal and transverse velocity components; x, y—longitudinal and transverse coordinates; T_0—stagnation temperature; P_b—base pressure; P_{ia}—static pressure; ρ_{ia}—density; M_{ia}—Mach number of stream (i = 1, 2, 3, 4), the number subscript indicates the region, the subscript *a* that the quantity belongs to the zero-gradient part of the stream; F—a quantity proportional to the stream function; P_{0j}, $F'_{F=0} = \bar{u}_j = u_j/u_{3a}$—respectively, total pressure and dimensionless longitudinal velocity on the branch line; x_*—mixing length; λ_{ia}—velocity coefficient; λ_j—velocity coefficient on the branch line; δ_1^{**}, δ_2^{**}—initial boundary-layer momentum thickness corresponding to deflection and after deflection of the flow; Θ_b—turning angle of the stream on passing through the shock; h—height of step; R—gas constant; $Cr^2 = 1\Big/\left(1+\frac{2}{\varkappa-1}\frac{1}{M^2}\right)$—Crocco number.

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